Fatigue Criterion for Ductile Outer Cylinders

From both torsion and triaxial fatigue tests on low-strength (120 to 150 ksi ultimate strength) steels conducted by Morrison, Crossland, and Parry[13] it is concluded that a shear criterion applies. Therefore, a shear theory of failure is assumed for outer rings made of ductile steel.

In order to formulate a fatigue relation, the semirange in shear stress and the mean shear stress are needed. These stresses are defined as

$$S_r = \frac{S_{max} - S_{min}}{2}$$

$$S_m = \frac{S_{max} + S_{min}}{2} \qquad (8a, b)$$

respectively.

The Goodman fatigue relation in terms of shear stresses is assumed. This relation is

$$\frac{S_r}{S_e} + \frac{S_m}{S_u} = 1, \text{ for } S_m \geqq 0 \quad ,$$

where S_e is the endurance limit in shear and S_u is the ultimate shear stress. For $S_u = 1/2\ \sigma_u$, where σ_u is the ultimate tensile stress, this relation can be rewritten as:

$$\frac{S_r}{S_e} \quad \frac{2S_m}{\sigma_u} = 1, \ S_m \geqq 0 \qquad (9)$$

The stresses S_r and S_m given by Equations (8a, b) can be calculated from elasticity solutions. In order to employ the fatigue relation (9) for general use, it is assumed that S_e can be related to S_u. This is a valid assumption as shown by Morrison, et al[13]. Referring to Reference (13), the ratio S_e/S_u can be established. Table 7 lists some fatigue data and results of calculation of S_e from Equation (9).

From Table 7 it is evident that fluid pressure contacting the material surface has a detrimental effect on fatigue strength; the endurance limit S_e for unprotected triaxial fatigue specimens is lower than that for torsional specimens. However, protection of the bore of triaxial specimens increases S_e under triaxial fatigue to a value equal that for torsional fatigue. Since in the high-pressure containers, outer cylinders are subject to interface contact pressures and not to fluid pressures, it is assumed that the latter data in Table 7 are applicable in the present analysis. Therefore, the following relation between S_e and σ_u is assumed:

$$S_e = \frac{1}{3}\sigma_u \qquad (10)$$

TABLE 7. TORSIONAL AND TRIAXIAL FATIGUE DATA ON VIBRAC STEEL[a]

Test	Stresses, psi				
	σ_u	S_r	S_m	S_e	S_e/σ_u
Torsion	126,000	43,700	0	43,700	0.347
	149,000	52,900	0	52,900	0.354
Triaxial (unprotected bore)	126,000	20,900	20,900	31,300[c]	0.248
	149,000	26,300	26,300	40,600	0.273
Triaxial[b] (protected bore)	126,000	26,500	26,500	45,900	0.363

(a) From Reference (13). Composition of this steel is 0.29 to 0.3 C, 0.14 to 0.17 Si, 0.64 to 0.69 Mn, 0.015 S, 0.013 P, 2.53 to 2.58 Ni, 0.57 to 0.60 Cr, 0.57 to 0.60 Mo.

(b) The bore of the cylindrical specimens was protected with a neoprene covering.

(c) S_e for the triaxial tests is calculated from Equation (9).